دانشکده مهندسی کامپیوتر

هوش مصنوعی و سیستم‌های خبره

---

تمرین تشریحی هفتم[1]

---

نام و نام خانوادگی - شماره دانشجویی . . . . . . . . . . . . . . . . . . . . . . . . . .

مدرس . . . . . . . . . . . . . . . . . . . . محمدطاهر پیله‌ور - سید صالح اعتمادی

طراحی و تدوین . . . . . . . . . . . . . . . . . . . مرسده ایرانی - غزاله محمودی

تاریخ انتشار . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ۲۶ آذر ۱۳۹۹

تاریخ تحویل گروه ۱ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . -

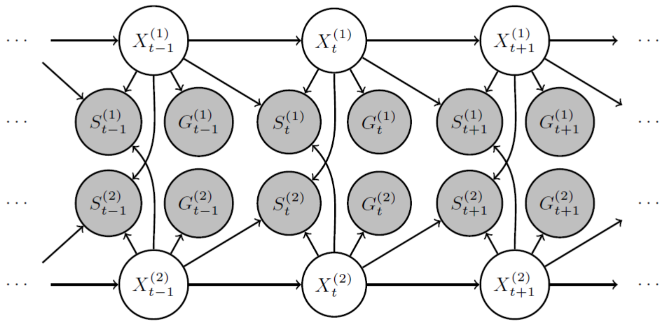
تاریخ تحویل گروه ۲ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ۳ دی ۱۳۹۹

[1] در طراحی این تمرین از منابع کورس CS188 دانشگاه برکلی استفاده شده است.

# ۱ Particle Filtering: Where are the Two Cars?

As before, we are trying to estimate the location of cars in a city, but now, we model two cars jointly, i.e. car i for $i \in \{1, 2\}$. The modified HMM model is as follows:

- $X^{(i)}$– the location of car $i$
- $S^{(i)}$– the noisy location of the car $i$ from the signal strength at a nearby cell phone tower
- $G^{(i)}$– the noisy location of car $i$ from GPS




| $d$ | $D(d)$ | $E_L(d)$ | $E_N(d)$ | $E_G(d)$ |
|---|---|---|---|---|
| -4 | 0.05 | 0 | 0.02 | 0 |
| -3 | 0.10 | 0 | 0.04 | 0.03 |
| -2 | 0.25 | 0.05 | 0.09 | 0.07 |
| -1 | 0.10 | 0.10 | 0.20 | 0.15 |
| 0 | 0 | 0.70 | 0.30 | 0.50 |
| 1 | 0.10 | 0.10 | 0.20 | 0.15 |
| 2 | 0.25 | 0.05 | 0.09 | 0.07 |
| 3 | 0.10 | 0 | 0.04 | 0.03 |
| 4 | 0.05 | 0 | 0.02 | 0 |

The signal strength from one car gets noisier if the other car is at the same location. Thus, the observation $S^{(i)}_t$ also depends on the current state of the other car $X^{(j)}_t$, $j \neq i$.

The transition is modeled using a drift model D, the GPS observation $G^{(i)}_t$ using the error model $E_G$, and the observation $S^{(i)}_t$ using one of the error models $E_L$ or $E_N$, depending on the car's speed and the relative location of both cars. These drift and error models are in the table above. **The transition and observation models are:**

$$P(X^{(i)}_t | X^{(i)}_{t-1}) = D(X^{(i)}_t - X^{(i)}_{t-1})$$

$$P(S^{(i)}_t | X^{(i)}_{t-1}, X^{(i)}_t, X^{(j)}_t) = \begin{cases} E_N(X^{(i)}_t - S^{(i)}_t), & \text{if } |X^{(i)}_t - X^{(i)}_{t-1}| \geq 2 \text{ or } X^{(i)}_t = X^{(j)}_t \\ E_L(X^{(i)}_t - S^{(i)}_t), & \text{otherwise} \end{cases}$$

$$P(G^{(i)}_t | X^{(i)}_t) = E_G(X^{(i)}_t - G^{(i)}_t).$$

Throughout this problem you may give answers either as unevaluated numeric expressions (e.g. $0.1 \cdot 0.5$) or as numeric values (e.g. $0.05$). The questions are decoupled.

۱.۱

Assume that at t = 3, we have the single particle $(X^{(1)}_3 = -1, X^{(2)}_3 = 2)$.

۱.۱.۱

What is the probability that this particle becomes $(X^{(1)}_4 = -3, X^{(2)}_4 = 3)$ after passing it through the dynamics model?

Answer= ........
Your Solution:

۲.۱.۱

Assume that there are no sensor readings at t = 4. What is the joint probability that the *original* single particle (from t = 3) becomes $(X^{(1)}_4 = -3, X^{(2)}_4 = 3)$ and then becomes $(X^{(1)}_5 = -4, X^{(2)}_5 = 4)$?

Answer= ........
Your Solution:

For the remaining of this problem, we will be using 2 particles at each time step.

۲.۱

At t = 6, we have particles $[(X^{(1)}_6 = 3, X^{(2)}_6 = 0), (X^{(1)}_6 = 3, X^{(2)}_6 = 5)]$. Suppose that after weighting, resampling, and transitioning from t = 6 to t = 7, the particles become $[(X^{(1)}_7 = 2, X^{(2)}_7 = 2), (X^{(1)}_7 = 4, X^{(2)}_7 = 1)]$.

۱.۲.۱

Suppose both cars' cell phones died so you only get the observations $G^{(1)}_7 = 2, G^{(2)}_7 = 2$. What is the weight of each particle?

| Particle | Weight |
|---|---|
| $(X_7^{(1)} = 2, X_7^{(2)} = 2)$ | |
| $(X_7^{(1)} = 4, X_7^{(2)} = 1)$ | |

Your Solution:

۳.۱

To decouple this question, assume that you got the following weights for the two particles.

| Particle | Weight |
|---|---|
| $(X_7^{(1)} = 2, X_7^{(2)} = 2)$ | 0.09 |
| $(X_7^{(1)} = 4, X_7^{(2)} = 1)$ | 0.01 |

What is the belief for the location of car 1 and car 2 at t = 7?

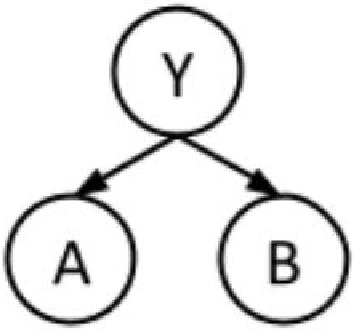
| Location | $P(X_7^{(1)})$ | $P(X_7^{(2)})$ |
|---|---|---|
| $X_7^{(i)} = 1$ | | |
| $X_7^{(i)} = 2$ | | |
| $X_7^{(i)} = 4$ | | |

Your Solution:

# ۲ Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y , A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| $B$ | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| $Y$ | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |




## ۱.۲

What are the maximum likelihood estimates for the tables P(Y ), P(A|Y ), and P(B|Y )?

| $Y$ | $P(Y)$ |
|---|---|
| 0 | |
| 1 | |

| $A$ | $Y$ | $P(A\|Y)$ |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |

| $B$ | $Y$ | $P(B\|Y)$ |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |

## ۲.۲

Consider a new data point (A = 1, B = 1). What label would this classifier assign to this sample?

۳.۲

Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for P(A|Y ) given Laplace Smoothing with k = 2.

| $A$ | $Y$ | $P(A\|Y)$ |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |